My first scientific paper

Week 2 — Select your project and tell about it

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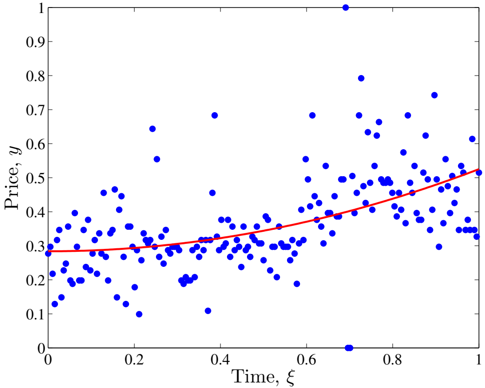
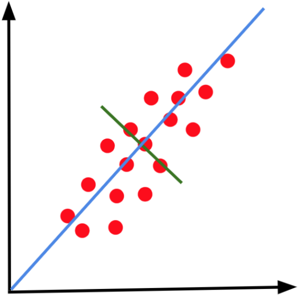
A simple model and its structure $\mathbf{a} \in \mathbb{B}^n$



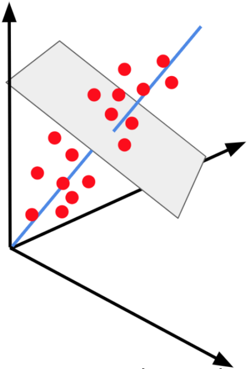
Regression model: $f = w_1 + w_2\xi^1 + w_3\xi^2 + \varepsilon(\xi)$, let $\mathbf{x} = [\xi^0, \xi^1, \xi^2]^\mathsf{T}$,
model to select from: $f = \mathbf{a} \odot \mathbf{w}^\mathsf{T}\mathbf{x}$,
optimal structure: $\hat{\mathbf{a}} = [1, \quad 0, \quad 1]^\mathsf{T}$,
optimal parameters: $\hat{\mathbf{w}} = [0.2839, \text{n/a}, 0.2412]^\mathsf{T}$.

Principal component analysis

PCA reconstructs dependency, reduces dimensionality



$2d \to 1d$



$3d \to 1d$

$$\boldsymbol{z} = \boldsymbol{W}^\top \boldsymbol{x} + \boldsymbol{\mu} + \varepsilon$$

PCA decomposes a set into deterministic $\boldsymbol{W}, \boldsymbol{\mu}$ and stochastic ε parts.

Principal Component Analysis by S. Mallick, 2018, LearnOpenCV

Model families

A model is a parametric family of functions,

$$\hat{y} = f(\hat{\mathbf{w}}, \mathbf{x}),$$

an element of a model family, given by some superposition,

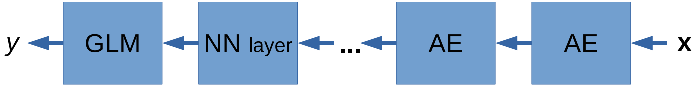
$$f = g_K \circ \cdots \circ g_1(\mathbf{w})(\mathbf{x}) \ni \mathfrak{F}.$$

An example is a superposition of linear maps (transformations) and non-linear monotonous (smooth) functions:

$$f(\mathbf{w}, \mathbf{x}) = \sigma_K \circ \mathbf{w}_K^{\mathsf{T}} \boldsymbol{\sigma}_{k-1} \circ \cdots \circ \boldsymbol{\sigma}_1 \mathbf{W}_1^{\mathsf{T}} \mathbf{x}.$$

The model parameters are treated as $\mathbf{w} = \mathbf{vec}(\mathbf{w}_K, \dots, \mathbf{W}_1)$

Neural network with stack of autoencoders



$$f = \underset{1\times 1_k}{\boldsymbol{w}^{\mathsf{T}}} \boldsymbol{z}_{k-1} \circ \boldsymbol{W}_{k-1}^{\mathsf{T}} \boldsymbol{z}_{k-2} \circ \cdots \circ \underset{n_2 \times 1}{\boldsymbol{W}_2^{\mathsf{T}} \boldsymbol{z}_1} \circ \underset{n_1 \times n}{\boldsymbol{W}_1^{\mathsf{T}}} \underset{n\times 1}{\boldsymbol{x}}$$

Neural network error

$$E_y = \big(y_i - f(\boldsymbol{x})\big)^2$$

Autoencoder reconstruction error

$$E_{\boldsymbol{x}} = \|\boldsymbol{x} - \boldsymbol{r}(\boldsymbol{z})\|^2$$

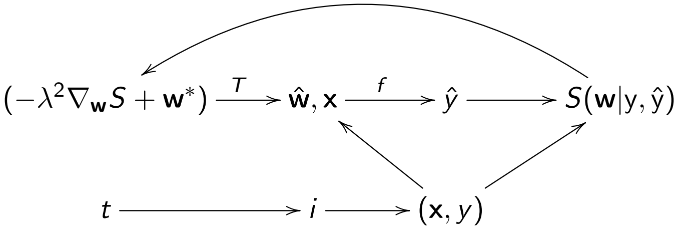
Types of autoencoders

PCA	skip block	metric	multi-linear
$\boldsymbol{W}^{\mathsf{T}}\boldsymbol{W} = \boldsymbol{I}_n$	$\boldsymbol{W} = \boldsymbol{I}_n$	$\boldsymbol{x}^{\mathsf{T}}\boldsymbol{W}\boldsymbol{x} \geqslant 0$	$\underline{\boldsymbol{W}}\boldsymbol{X}$

Autoencoder transform: $\boldsymbol{z} = \big(1 + \exp(-\boldsymbol{W}^{\mathsf{T}}\boldsymbol{x} + \boldsymbol{b})\big)^{-1}$

Reconstruction decoder: $\hat{\boldsymbol{x}} = \boldsymbol{r}\big(\boldsymbol{z}(\boldsymbol{x})\big)$

The simplest problem statement: discriminative



$$\hat{\mathbf{w}} = \arg\min S(\mathbf{w}|y, f).$$