Kuznetsov M.P., Strijov V.V. Partial Orders Combining for the Object Ranking Problem // Conference of the International Federation of Operational Research Societies, 2014 : 157.

■ HA-15

Thursday, 8:30-10:00 - Room 125

Revenue Management Application and Theory

Stream: Revenue Management II
Invited session
Chair: *Darius Walczak*

1 - Challenges in RM & Pricing Optimization of Product-Resource Networks
Darius Walczak

We review optimization challenges in product-resource networks found in revenue management and pricing applications. Average network consists of thousands of products where each product consumes a finite number of resources, and the objective is to select one of several price points for each product so that expected revenue is maximized. Due to dimensionality and stochasticity in the problem, real-life software has to rely on near-optimal controls. We present some of these approaches. We also revisit calculating other business metrics, such as expected demand, for a given solution.

2 - Shaping Demand to Match Anticipated Supply
Anant Balakrishnan, Sifeng Lin, Yusen Xia

Firms can exploit their information on inbound supplies to better match demand with anticipated supply through dynamic pricing. We develop an economic model to address short-run demand shaping decisions for vertically differentiated products, i.e., to determine the prices for high and low quality products in each period so as to dynamically segment the market and maximize profits. We identify properties of the optimal price and sales trajectories, and assess the benefit of dynamic pricing versus myopic or sequential pricing approaches.

3 - An Efficient Pricing Method to Determine the Network Value of Influentials in Social Networks
Evren Guney, Volkan Çakır, Irem Düzdar, Abdullah Ozdemir

Companies use social networks to benefit from word-of-mouth marketing by influentials. Most of the previous studies focus on how to maximize the number of individuals reached starting from an initial set of influentials. However, many companies are focused on the total revenue. Hence, a modified objective function that maximizes total revenue, instead of the number of individuals, is proposed. An efficient pricing method to determine the network value of customers is developed and influence maximization is studied from the aspect of revenue maximization and tested on certain real-life data.

4 - A Model for Competition in Network Revenue Management
Nishant Mishra

We study a model of competition in network revenue management where multiple risk-averse players compete to satisfy uncertain consumer demand. For a linear inverse demand function, and for a symmetric game, we can come-up with closed form expressions for equilibrium quantities and prices, and we also establish some monotonicity properties. We then numerically study asymmetric competition to generate further insights. For instance, we find that asymmetry with respect to risk aversion has the same effect as higher demand uncertainty for the more risk averse competitor.

■ HA-16

Thursday, 8:30-10:00 - Room 127

Categorical Data Analysis and Preference Aggregation

Stream: Intelligent Optimization in Machine Learning and Data Analysis
Invited session
Chair: *Michael Doumpos*

1 - Partial Orders Combining for the Object Ranking Problem
Mikhail Kuznetsov, Vadim Strijov

We propose a new method for the ordinal-scaled object ranking problem. The method is based on the combining of partial orders corresponding to the ordinal features. Every partial order is described with a positive cone in the object space. We construct the solution of the object ranking problem as the projection to a superposition of the cones. To restrict model complexity and prevent overfitting we reduce dimension of the superposition and select most informative features. The proposed method is illustrated with the problem of the IUCN Red List monotonic categorization.

2 - An Interactive Approach for Multicriteria Selection Problem
Anil Kaya, Ozgur Ozpeynirci, Selin Ozpeynirci

In this study, we work on multiple criteria selection problem. We assume a quasiconcave utility function that represents the preferences of the decision maker (DM). We generate convex cones based on the pairwise comparisons of DM. Then, we build a mathematical model to determine the minimum number of pairwise comparisons required to eliminate all alternatives but the best one. Using the properties of the optimal cones and the pairwise comparisons, we develop an interactive algorithm. We conduct computational experiments on randomly generated instances.

3 - Data-Driven Robustness Analysis for MCDA Preference Disaggregation Approaches
Michael Doumpos, Constantin Zopounidis

Preference disaggregation (PD) is involved with inferring multicriteria decision models from decision examples. The robustness of models and recommendations obtained through PD methods, has attracted much interest. Previous research has mostly focused on uncertainties related to preferential parameters of decision models. In the context of PD, however, the data used to infer the model also affect the robustness of the results. In this presentation we discuss this issue and present ways to enhance existing robust MCDA techniques in a data-driven context.

■ HA-17

Thursday, 8:30-10:00 - Room 005

Second-Order Conic Optimization

Stream: Interior Point Methods and Conic Optimization
Invited session
Chair: *Jacek Gondzio*

1 - Mixed-Integer Second-Order Conic Optimization (MISOCO): Disjunctive Conic Cuts and Portfolio Models
Tamás Terlaky

The use of integer variables naturally occurs in Second Order Conic Optimization problems, just as in linear and nonlinear optimization. Thus, the need for dedicated MISOCO algorithms and software is evident. This talk gives some insight into the design of Disjunctive Conic Cuts (DCCs) for mixed-integer CLO problems, and into the complexity of identifying disjunctive conic cuts. The novel DCCs may be used to develop Branch-and-Cut algorithms for MISOCO problems. Preliminary computational experiments by solving classes of MISOCO Portfolio Selection problems show the power of the DCC approach.

2 - Interior-Point Methods within Algorithms for Mixed-Integer Second-Order Cone Programming
Hande Benson

Second-order cone programming problems (SOCPs) have been well-studied in literature, and computationally efficient implementations of solution algorithms exist. In this talk, we study an extension: mixed-integer second-order cone programming problems (MISOCPs). Our focus is on designing an algorithm for solving the underlying SOCPs as smooth, convex NLPs, while using primal-dual regularization to introduce warmstarting and infeasibility detection capabilities. We present numerical results obtained using the Matlab-based optimization package, MILANO.

Partial Orders Combining for Objects Ranking Problem

M. P. Kuznetsov, V. V. Strijov

Moscow Institute of Physics and Technology

20th Conference of the International Federation of Operational Research Societies, Barcelona, 2014

Object ranking with monotone constraints

There given a sample of m objects,

$$\mathfrak{D} = \{(\mathbf{x}_i, y_i)\}_{i=1}^{m},$$

where

- $x_{ij} \in X_j$ belongs to a partially ordered set X_j,
- $y \in Y$ belongs to a response variable ordered set of values Y.

The goal: to construct a mapping

$$f: \quad X_1 \times ... \times X_n \to Y.$$

Partially ordered set of feature values

The set X_j with a given partial order relation $\succeq$ with the following properties:

- reflexivity, $\forall a \in X_j \ (a \succeq a)$,
- antisymmetry, $\forall a, b \in X_j$, $(a \succeq b) \wedge (b \succeq a) \Rightarrow (a = c)$,
- transitivity, $\forall a, b, c \in X_j \ (a \succeq b) \wedge (b \succeq c) \Rightarrow (a \succeq c)$.

Partial order matrix

A partial order matrix $\mathbf{Z}_j$ for the sample $\mathfrak{D}$ and the set X_j describes binary relation between each pair of the sample elements,

$$\mathbf{Z}_j(i,k) = \begin{cases} 1, & \text{if} \quad x_{ij} \succeq x_{kj}, \\ 0, & \text{if} \quad x_{ij} \not\succeq x_{kj}. \end{cases}$$

An example of a partial order relation graph and a corresponding matrix $\mathbf{Z}_j$:

$$\begin{tikzcd} x_1 \arrow[r] & x_2 \arrow[r] \arrow[dr] & x_3 \\ & & x_4 \end{tikzcd} \qquad \mathbf{Z}_j = \begin{pmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}.$$

Rank correlation in terms of partial order matrix

Let $\mathbf{r}_1$, $\mathbf{r}_2$ be the two rankings. $\mathbf{Z}_1, \mathbf{Z}_2$ are partial order matrices, corresponding to $\mathbf{r}_1$, $\mathbf{r}_2$.

- Kendall rank correlation:

$$\tau(\mathbf{r}_1, \mathbf{r}_2) \propto \sum_i \sum_k ([r_{1i} < r_{1k}] \neq [r_{2i} < r_{2k}]),$$

$$\tau(\mathbf{r}_1, \mathbf{r}_2) \propto \sum_i \sum_k (\mathbf{Z}_1(i,k) - \mathbf{Z}_2(i,k))^2 \, .$$

- Spearman rank correlation:

$$\rho_s(\mathbf{r}_1, \mathbf{r}_2) \propto \sum_{i=1}^{m} (r_{1i} - r_{2i})^2,$$

$$\rho_s(\mathbf{r}_1, \mathbf{r}_2) \propto \sum_i \left(\sum_k \mathbf{Z}_1(i,k) - \mathbf{Z}_2(i,k) \right)^2 \, .$$

Voting rules in terms of partial order matrix

- Kemeny rule: $\mathbf{Z}_1, ..., \mathbf{Z}_n$ are the voting matrices, $\hat{\mathbf{Z}} \in \mathrm{LO}$ is an optimal voting:

$$\hat{\mathbf{Z}} = \min_{\mathbf{Z} \in \mathrm{LO}} \sum_{j=1}^{n} \sum_{i,k} (\mathbf{Z}(i,k) - \mathbf{Z}_j(i,k))^2.$$

- Condorcet voting rule: regard the sum of partial order matrices,

$$\hat{\mathbf{Z}} = \sum_{j=1}^{n} \mathbf{Z}_n,$$

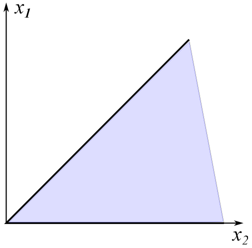
and find the «winner» using e.g. a Ranking Pairs method.

A partial order cone

For each set X_j define a cone $\mathcal{X}_j$ in a space $\mathbb{R}^m_+$:

$$\mathcal{X}_j = \{\boldsymbol{\chi}_j \in \mathbb{R}^m_+ | \quad x_{ij} \succeq x_{kj} \to \chi_{ij} \geq \chi_{kj} \quad \forall i, k = 1, ..., m\}.$$

Example for $x_2 \succeq x_1$:



Vector decomposition theorem

A vector $\boldsymbol{\chi}$ belonging to the cone $\mathcal{X}$ can be represented as the nonnegative combination of cone generators $\mathbf{z}_k$,

$$\boldsymbol{\chi} = \sum_{k=1}^{m} \lambda_k \mathbf{z}_k, \quad \lambda_k \geq 0,$$

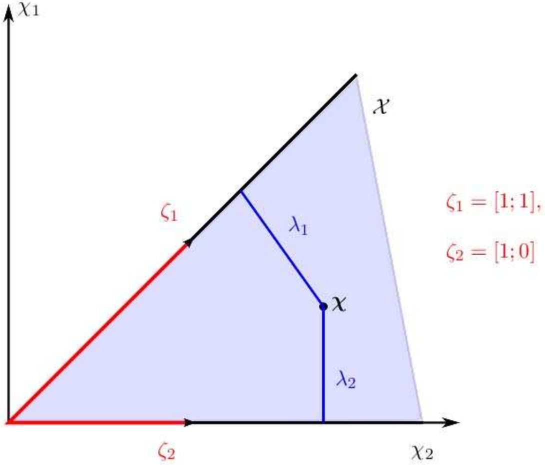
where $\mathbf{z}_k$ is also a column of a matrix $\mathbf{Z}$,

$$\mathbf{z}_k(i) = \begin{cases} 1, & \text{if} \quad x_i \succeq x_k, \\ 0, & \text{if} \quad x_i \not\succeq x_k, \end{cases}$$

and the decomposition is unique.

Vector decomposition illustration

$$\mathbf{Z} = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}, \quad \chi = \sum_{k=1}^{m} \lambda_k \mathbf{z}_k, \quad \lambda_k \geq 0.$$



Set of solutions for the supervsied problem

- Let target estimation $\hat{\mathbf{y}}$ of the vector $\mathbf{y}$ belong to a cones superposition, $\hat{\mathbf{y}} \in \sum\limits_{j=1}^{n} \mathcal{X}_j$.
- Then $\hat{\mathbf{y}}$ can be expressed as the non-negative combination of the cones generators:

$$\hat{\mathbf{y}} = \sum_{j=1}^{n} w_j \mathbf{Z}_j \boldsymbol{\lambda}_j, \quad \boldsymbol{\lambda}_j \geq 0, \quad \|\boldsymbol{\lambda}_j\|_1 = 1.$$

$$\hat{\mathbf{y}} = w_1 \underbrace{\begin{array}{c} \begin{array}{cccc} \lambda_{11} & \lambda_{12} & \dots & \lambda_{1m} \end{array} \\ \begin{pmatrix} 1 & 1 & \dots & 0 \\ 0 & 0 & \dots & 1 \\ \dots & \dots & \dots & \dots \\ 1 & 0 & \dots & 1 \end{pmatrix} \end{array}}_{\mathbf{Z}_1} + w_2 \underbrace{\begin{array}{c} \begin{array}{cccc} \lambda_{21} & \lambda_{22} & \dots & \lambda_{2m} \end{array} \\ \begin{pmatrix} 1 & 0 & \dots & 1 \\ 0 & 0 & \dots & 0 \\ \dots & \dots & \dots & \dots \\ 0 & 1 & \dots & 1 \end{pmatrix} \end{array}}_{\mathbf{Z}_2}$$

Parameter estimation

- Optimal parameters $\hat{\mathbf{w}}, \hat{\boldsymbol{\lambda}}_j$ minimize the loss function:

$$(\hat{\mathbf{w}}, \hat{\boldsymbol{\lambda}}_j) = \arg\min_{\mathbf{w}, \boldsymbol{\lambda}_j} \|\mathbf{y} - \hat{\mathbf{y}}\|, \quad \hat{\mathbf{y}} = \sum_{j=1}^{n} w_j \mathbf{Z}_j \boldsymbol{\lambda}_j.$$

- Define the vector of objects weights $\boldsymbol{\lambda}$, such that $\boldsymbol{\lambda}_j = \boldsymbol{\lambda}$. Then

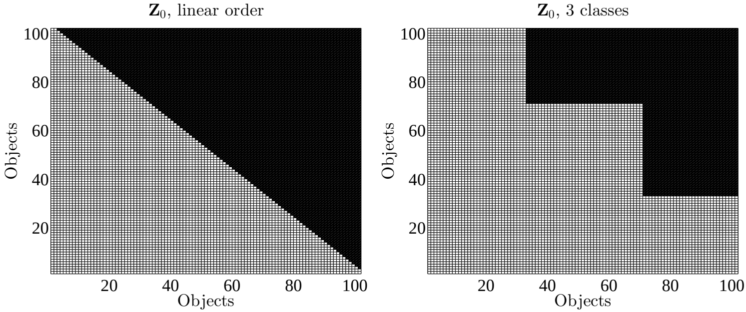
$$\hat{\mathbf{y}} = \sum_{j=1}^{n} w_j \mathbf{Z}_j \boldsymbol{\lambda} = \hat{\mathbf{Z}} \boldsymbol{\lambda},$$

where $\hat{\mathbf{Z}} = w_1 \mathbf{Z}_1 + \ldots + w_n \mathbf{Z}_n$ is a matrix of a "fuzzy"preference relation,

$$\hat{\mathbf{Z}}(i, k) \propto \hat{p}(\mathbf{x}_i \succeq \mathbf{x}_k).$$

Feature weights estimation

- $\mathbf{Z}_0$ is a partial order matrix for the expert-given vector of target variables $\mathbf{y}$.



- Instead of full optimization, estimate parameters $\mathbf{w}$ such that:

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}} \|\mathbf{Z}_0 - \sum_{j=1}^{n} w_j \mathbf{Z}_j\|.$$

2-step algorithm of parameter estimation

1. Having matrices $\mathbf{Z}_1, \ldots, \mathbf{Z}_n$, estimate feature weights $\hat{\mathbf{w}}$ and a fuzzy preference relation matrix $\hat{\mathbf{Z}}$,

$$\hat{\mathbf{w}} = \arg\min_{\mathbf{w}} \|\mathbf{Z}_0 - \sum_{j=1}^{n} w_j \mathbf{Z}_j\|.$$

2. Having matrix $\hat{\mathbf{Z}}$, estimate object weights $\hat{\boldsymbol{\lambda}}$ and a target ranking $\hat{\mathbf{y}}$,

$$\hat{\boldsymbol{\lambda}} = \arg\min_{\boldsymbol{\lambda}} \|\mathbf{y} - \hat{\mathbf{Z}}\boldsymbol{\lambda}\|.$$

Example: The IUCN Red List categorization

- The main purpose of the IUCN Red List is to catalogue those animals and plants that are facing higher risk of extinction.
- All species should be categorized as
    1. **EW** — extinct in the wild,
    2. **CR** — critically endangered,
    3. **EN** — endangered,
    4. **VU** — vulnerable,
    5. **NT** — near threatened,
    6. **LC** — least concern.

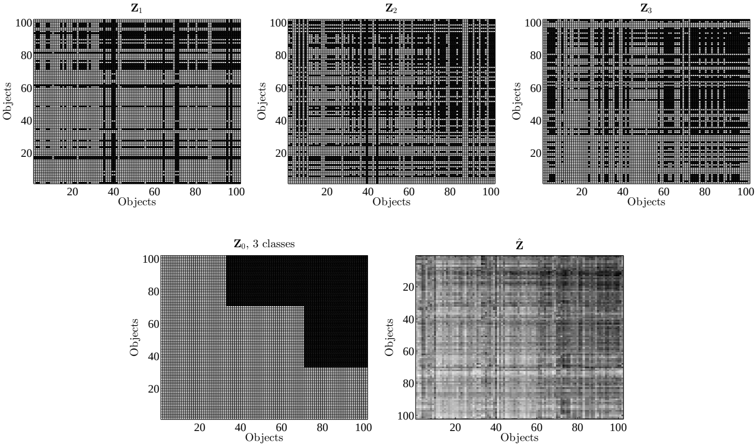
IUCN species description

- Each species is described by the set of ordinal features: **PS** — population size, **AS** — area square, **GD** — genetic diversity, etc.
- An initial categorization is also given.

Species	**PS**	**AS**	**GD**	Category, **y**
Green Sturgeon	2	2	0	**EW**
Lagoda Whitefish	0	2	1	**CR**
Long-finned Charr	3	1	0	**EN**
Polar Bear	3	3	0	**NT**
Sandpiper	2	1	0	**EN**
Shizophragma	1	1	1	**EW**
Tropical Lichens	2	1	1	**LC**

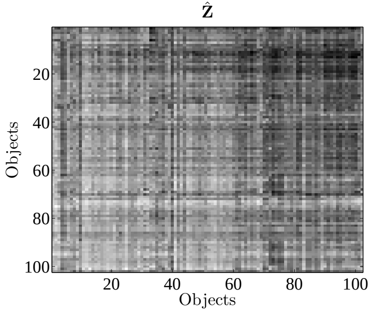
IUCN example, relation matrix estimation

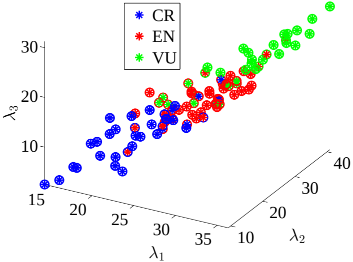
Input: vector $\mathbf{y}$, matrix $\mathbf{Z}_0$, feature matrices $\mathbf{Z}_1, \ldots, \mathbf{Z}_n$.
Output: matrix $\hat{\mathbf{Z}}$.



IUCN example, object categories estimation

Input: relation matrix $\hat{\mathbf{Z}}$, vector $\mathbf{y}$.
Output: estimated classification $\hat{\mathbf{y}}$.





Conclusion and further research

- We proposed a ranking algorithm based on partial orders combining.
- The algorithm based on estimation of a fuzzy preference relation matrix.
- The set of possible solutions is a superposition of partial order cones.
- The further investigation now is to propose a regularization method to choose most informative objects.